Analytic Geometry – How to Dilate Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Open the file **How\_to\_Dilate.ggb.** Orient yourself with the file. Observe in the top left corner what *s, c,* & *d*  are, as well as where the sliders are.

1. When $s=2$, what is the relationship between the coordinates of the image and the pre-image? Adjust the slider so that $s=3, s=1, s= \frac{1}{2}, s=0, s=-2, s=-3$. Can you make a conjecture about the relationship between the scale factor *s* and the coordinates of the image?

2. Move the c and *d* sliders so that the center of dilation moves around. Does your conjecture from question 1 hold?

3. What changed from question 1 to question 2? Can you refine your conjecture from question 1?

4. Use the sliders to move the center of dilation D to (2, -1) and the scale factor to $s=2$. Right click anywhere in blank space until you see a menu with **Graphics** at the top. Click on **Axes**. Now, click on the “Show new coordinate plane” box. Describe what you see. What happened to the coordinate plane? What does the point O represent?

5. In question 5, you pretended that D was the origin; if it was, we could use the conjecture from question 3. However, it is not. What geometric transformation could you use to move the entire coordinate plane (along with D and the triangles) so that O is the origin again? Drag the coordinate plane accordingly.

6. Click the “Show first auxiliary triangle” box. (Note: “auxiliary” describes something that helps or supports.) What happened to $∆ABC$? Does this agree with your answer to question 5?

7. Click the “Show second auxiliary triangle.” Describe specifically what happens.

8. What geometric transformation would transform the second auxiliary triangle into $∆A^{'}B^{'}C^{'}?$ Is there anything significant about this? (Ring any bells?)

9. Click the “Show third auxiliary triangle” box. Were you correct in 8?

10. Review your answers to the previous question and complete the following table to create a rule for dilating any point (x,y) from any center of dilation (c, d) by any scale factor *s*.

HINT: In questions 4 – 8, $c=2, d=-1, s=2$. Three transformations occurred to get $∆ABC$ to $∆A'B'C'$.

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| **Mathematical Representation** | **Transformation Occurring** | **In My Own Words****(What’s happening and why?)** |
| (x,y) 🡪 ( , ) |  |  |
| ( , ) 🡪 ( , ) |  |  |
| ( , ) 🡪 ( , ) |  |  |

10. Click the third tool on the tool bar. Select **Line through Two Points.** Click on point A, then A’. Repeat for B & B’ and C & C’. Write down your observation about the three lines.

11. Use the sliders to change *c, d,* & *s*. Move the vertices A, B, and C around as much as you like. Can you make a conjecture related to the three lines? Be specific.

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1. When $s=2$, what is the relationship between the coordinates of the image and the pre-image? Adjust the slider so that $s=3, s=1, s= \frac{1}{2}, s=0, s=-2, s=-3$. Can you make a conjecture about the relationship between the scale factor *s* and the coordinates of the image?

The coordinates of the image are just the coordinates of the pre-image multiplied by the scale factor.

2. Move the c and *d* sliders so that the center of dilation moves around. Does your conjecture from question 1 hold?

No, the conjecture no longer holds.

3. What changed from question 1 to question 2? Can you refine your conjecture from question 1?

The center of dilation was no longer the origin. The conjecture from question 1 can be refined to hold only for a dilation with center (0,0).

4. Use the sliders to move the center of dilation D to (2, -1) and the scale factor to $s=2$. Right click anywhere in blank space until you see a menu with **Graphics** at the top. Click on **Axes**. Now, click on the “Show new coordinate plane” box. Describe what you see. What happened to the coordinate plane? What does the point O represent?

Coordinate plane was translated until D was the origin. O is the origin of the original plane.

5. In question 5, you pretended that D was the origin; if it was, we could use the conjecture from question 3. However, it is not. What geometric transformation could you use to move the entire coordinate plane (along with D and the triangles) so that O is the origin again? Drag the coordinate plane accordingly.

Translate one unit up and two units to the left.

6. Click the “Show first auxiliary triangle” box. (Note: “auxiliary” describes something that helps or supports.) What happened to $∆ABC$? Does this agree with your answer to question 5?

Triangle ABC was translated one unit up and two units to the left. Yes, this agrees this with the previous answer.

7. Click the “Show second auxiliary triangle.” Describe specifically what happens. Dilation from (0,0) with s= 2

8. What geometric transformation would transform the second auxiliary triangle into $∆A^{'}B^{'}C^{'}?$ Is there anything significant about this? (Ring any bells?)

Translate two units down and one unit to the right; this is the opposite of what we did with the coordinate plane

9. Click the “Show third auxiliary triangle” box. Were you correct in 8?

Yes. The two triangles lay directly in the same location.

10. Review your answers to the previous question and complete the following table to create a rule for dilating any point (x,y) from any center of dilation (c, d) by any scale factor *s*.

HINT: In questions 4 – 8, $c=2, d=-1, s=2$. Three transformations occurred to get $∆ABC$ to $∆A'B'C'$.

|  |  |  |
| --- | --- | --- |
| **Mathematical Representation** | **Transformation Occurring** | **In My Own Words****(What’s happening and why?)** |
| (x,y) 🡪 ( x - c , y - d ) | Translation | Shifting coordinate plane with D as origin to *real* origin |
| ( x-c , y-d ) 🡪 ( s(x-c) , s(y-d) ) | Dilation | Dilating from origin (because we know how to do that) |
| (s(x-c), s(y-d))🡪(s(x-c)+c, s(y-d)+d) | Translation | Putting the coordinate plane back |

10. Click the third tool on the tool bar. Select **Line through Two Points.** Click on point A, then A’. Repeat for B & B’ and C & C’. Write down your observation about the three lines.

The three lines all intersect at the center of dilation.

11. Use the sliders to change *c, d,* & *s*. Move the vertices A, B, and C around as much as you like. Can you make a conjecture related to the three lines? Be specific.

Lines through the corresponding vertices of a pre-image and image of a dilation intersect (uniquely) at one point, the center of dilation.