Activity 2 – Useful Inverses

1. When are inverses useful? Let’s say we have the system of equations below:

3x+5y=2

7x-2y=3

Here, mention the two methods you used to solve such a system of equations – solving and plugging in, and adding/subtracting the equations from each other.

We can represent this system of equations by a single matrix equation of the form Ax=b as follows:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 3 | 5 | times | X | = | 2 |
| 7 | -2 |  | Y |  | 3 |

Here A is called the coefficient matrix, x the variable matrix, and b the constant matrix. Do matrix multiplication below to make sure that this equation is true.

This web applet cannot handle variables, so this calculation must be done by hand.

|  |  |  |
| --- | --- | --- |
| 3x+5y | = | 2 |
| 7x-2y |  | 3 |

1. Use the matrix calculator (http://www.mathsisfun.com/algebra/matrix-calculator.html) to find the inverse of A. What happens if we multiply both sides of the equation on the left by A?

Have a discussion why “on the left” is a very important statement. Then note that the column vectors are equal, so each entry must be the same and we actually solved for both variables in this case.

|  |  |  |
| --- | --- | --- |
| X | = | .4634146 |
| Y |  | .1219512 |

After this, take the time to demonstrate finding the inverse by hand, demonstrating the process and how it is similar to solving a system of equations. If you want, use http://matrixcalc.org/en/ to have them observe the steps the computer uses to find the inverse by having them click “details”.

1. Now that we’ve looked at inverses, let’s check out determinants.

If they didn’t notice the relationship between inverses and nonzero determinants, mention it now.

What is the relationship between det(A) and det(A-1)?

det(A)\*det(A-1)=1

Now you should show how to actually calculate the determinant. Begin by showing for 2x2. Then show for larger matrices via the repeating columns and/or expansion of minors methods. Explain that this is how the computer does it as well, and there is no reason not to use technology for large matrices. You may again want to use http://matrixcalc.org/en/ to have your students see how the computer calculates a determinant. This can lead to a discussion about why making a matrix upper-triangular is useful.