Activity 1 – Beetle Survey

Materials: Pencil and paper, <http://www.mathsisfun.com/algebra/matrix-calculator.html>

Note: You may want to do this over multiple days depending on your class length. A good place to stop would be after #7. You could collect the handouts so that you can see student progress, briefly correct them or give feedback, and then return them to the students next class so that they can learn from their work and move on.

Scenario: Liz is a scientist studying beetle population in Georgia beaches. When she surveyed last Saturday she collected 153 female beetles and 92 male beetles. Of the 153 female beetles she collected 71 were black, 34 were gray, and 48 were white. Of the 92 male beetles she collected 43 were black, 28 were gray, and 21 were white.

Day 1

1. Record this data in the table below.

|  |  |  |
| --- | --- | --- |
|  | Female | Male |
| Black | 71 | 43 |
| Gray | 34 | 28 |
| White | 48 | 21 |

1. Often we leave off the labels since we know what it is talking about. Fill in the matrix below with the information, but without labels.

|  |  |
| --- | --- |
| 71 | 43 |
| 34 | 28 |
| 48 | 21 |

Talk about how this is actually a 3x2 matrix. Matrix because it is a rectangular array of values, 3x2 because it has 3 rows and 2 columns (not counting labels).

We often want to talk about specific *entries* in a matrix. In general we could write a 3x2 matrix as

|  |  |
| --- | --- |
| a1,1 | a1,2 |
| a2,1 | a2,2 |
| a3,1 | a3,2 |

What is the element a3,1 in the matrix you filled out above? What is the significance of the subscripts? What is the significance of the number in a3,1?

48. row, column. Liz collected 48 white female beetles on Saturday.

1. Liz also collected beetles on Sunday. The matrix of values is given below.

|  |  |
| --- | --- |
| 103 | 142 |
| 76 | 82 |
| 52 | 15 |

How many male beetles did she find that were gray? 82

Can we construct a matrix that represents the total data from the two collections? How would we go about doing that?

Yes, by adding the two matrices together.

Expect students to add the numbers by hand and then make a new matrix. Hopefully some of them will think of adding the matrices together.

1. One way to do this is called matrix addition. Open up the matrix calculator at <http://www.mathsisfun.com/algebra/matrix-calculator.html> , set the matrices to the appropriate sizes, and enter Saturday’s data in matrix A and Sunday’s data in matrix B. Then press the A+B button. How did it get the resulting matrix? Does it agree with your method? Write down the answer here.

It adds each entry with the corresponding entry in the other matrix.

|  |  |
| --- | --- |
| 174 | 185 |
| 110 | 110 |
| 100 | 36 |

Here you should state specifically how matrices add and subtract from each other. Then talk about how multiplication by scalars work. You may find it useful to have them change c, press cA, and try to figure out what happened.

1. Liz also had to keep track of where she found these beetles since she sampled at two sites, Tybee Island and Sapelo Island. In Tybee she found 20% of the black beetles, 45% of the gray beetles, and 35% of the white beetles. Using this information (and decimal representations of these percents) create a 2x3 matrix below that describes this data completely. Then label the rows and the columns.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Black | Gray | White |
| Tybee Island | .2 | .45 | .35 |
| Sapelo Island | .8 | .55 | .65 |

Expect students to be unsure what to do with the second row. Let them discuss among themselves if necessary – this should guarantee that they have a good understanding of percents.

1. Can you write a matrix whose rows are the collecting sites and columns are the amount of each gender found over there over the weekend?? Write your matrix here. (Hint: Try to figure out one entry at a time.)

|  |  |
| --- | --- |
| 119.3 | 99.1 |
| 264.7 | 231.9 |

The fact that there are decimals here allows for a good discussion on the significance of numbers and rounding. For instance, it is unlikely that she found .1 of a beetle. It is much more likely that one of the percents was rounded to be a nice number.

We can use the matrix in #5 together with the matrix you wrote for #4 to get this matrix. Enter your matrix from #5 for matrix A and the one from #4 for matrix B (You may have to change the size of your matrices). Then press the AB button to multiply the two matrices together. This should match your matrix above.

This would be a good chance to contrast matrix addition with multiplication, as the process for multiplication is not obvious like addition. Then define the matrix multiplication process formally and talk about the need for units of the columns to be the same as the rows. You can also have them experiment multiplying different matrices together to see what size the resulting matrix is.

Furthermore, the fact that this assumes even distribution across all colors should be discussed in detail with your students, since technically you could have found all the black male beetles at Tybee, for instance, and the matrix multiplication does not account for that.

1. Again, make your matrix A the one from #5 and matrix B the one from #4. What happens if you press A⬄B and then AB? Note that this is the same as multiplying BA.

You get a 3x3 matrix.

This is an example where AB and BA are different size matrices, so they are clearly not the same.

Also discuss how due to units, BA really makes no sense at all, while AB has meaning. Perhaps make them try to extrapolate a meaning for BA.

What if A and B were square matrices? Create your own and experiment with AB and BA. Are the results the same?

Generally, no.

Day 2

1. Open up the matrix calculator at <http://www.mathsisfun.com/algebra/matrix-calculator.html>. For this exercise, you will create 5 square matrices. Below, record the matrix. Then enter it as matrix A and press det(A). Record the value in the second column. Now press inv(A) to get the inverse matrix A-1. Record whether or not the matrix calculator gave you a matrix or if the inverse does not exist in the third column. Finally, if the inverse does exists, press the “to B” button. Now multiply A and B together and write down the matrix AB in the last column (leave it blank if there is no inverse).

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

What do you notice about the matrices in the fourth column?

They are all identity matrices.

Here introduce the idea of inverse matrices and when they can occur. You may want to note that the inverse matrix will only have integers in it if the determinant was +/-1. Note the parallel between inverse matrices and fractions, then ask them for a parallel of 0 as a matrix.

Even though I have them use the “determinant” button here, it is merely to see that the determinant is 0 when an inverse does not exist. Determinants will be discussed in the next activity.